FUSION REACTIVITY ENHANCEMENT DUE TO ION TAIL
FORMATION BY BEAM AND ION CYCLOTRON HEATING

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Mitsuru YAMAGIWA, Tomonori TAKIZUKA and Yasuaki KISHIMOTO

日本原子力研究所
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FUSION REACTIVITY ENHANCEMENT DUE TO ION TAIL FORMATION
BY BEAM AND ION CYCLOTRON HEATING

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Fusion reactivity enhancement due to ion tail formation by NBI (neutral-beam (D\textsuperscript{+}) injection) and second harmonic ICH (ion (D\textsuperscript{+}) cyclotron heating) in a 50:50 D-T plasma is investigated on the basis of local Fokker-Planck calculation. Deformation of the deuteron velocity distribution function is examined analytically and comparison is made with numerical results by using a Fokker-Planck code. The reactivity is given by \(\langle \sigma v \rangle = \int d\vec{v} f(\vec{v}) G(\vec{v})\), where \(f\) is the deuteron distribution and \(G\) is called '\(\sigma v\)-function' averaged by the isotropic triton distribution. The profile of the integrand typically presents two humps in velocity space for the beam-induced tail. This results in large reactivity enhancement for high energy beam injection. For the case of the RF (radio frequency)-induced tail, a 'wing' rather than a hump is formed. ICRF (ion cyclotron range of frequency) waves well couple with the beam-induced tail ions and enhance the reactivity, especially when the beam is injected perpendicularly to the magnetic field. The possibility that the combined effects of NBI and second harmonic ICH can exceed the each effect on the reactivity enhancement in the efficiency is indicated.

Keywords: Fusion Reactivity Enhancement, Ion Tail Formation,
Neutral Beam Injection, Second Harmonic Ion Cyclotron Heating,
Fokker-Planck Equation, Quasi-Linear RF Diffusion, D-T Plasma
ビームおよびイオンサイクロトロン加熱によるイオンテイル形成に基づく核融合反応特性の増大

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（1987年4月2日受理）

50:50重水素-三重水素プラズマにおけるNBI（中性粒子（重水素）ビーム入射）および第2高調波ICH（重水素イオンサイクロトロン加熱）によるイオンテイル形成に基づく核融合反応特性の增大を局所的なフォッカー・プランク方程式を用いた計算により調べる。重水素イオンの速度分布関数の変形を解析的に吟味し、フォッカー・プランクコードを用いた数値解析結果との比較を行う。反応特性は＜σν＞=∫dνf(ν)G(ν)により与えられる。ここで、fは重水素イオンの速度分布関数、Gは等方的な三重水素イオンの速度分布について平均されたσνー関数と呼ばれる。被積分関数のプロファイルはビームテイルに関して典型的には速度空間上で2つの瓣を呈する。これにより高エネルギーのビーム入射に対しては大きな反応特性の増大が得られる。RF（高周波）によってつくれられるテイルの場合は瓣というより“翼”が形成される。ICRF（イオンサイクロトロン周波数帯）波は、特にビームが磁場に対して垂直に入射されたときビームテイルイオンとのカップリングが良く、反応特性を増大させる。NBIと第2高調波ICHとの複合効果は反応特性増大の効率においてのおのおのを凌駕する可能性が示される。
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1. INTRODUCTION

High energy ions play important roles in a fusion reactor of magnetic confinement system, for example, in the case of current drive by neutral-beam injection (NEI) [1] and beam-driven reactor operation [2] such as two-energy-component torus (TCT) and counterstreaming-ion torus (CIT). The concept of TCT was proposed by Dawson et al. [3]. It was found that the fusion reactivity can be enhanced owing to the presence of the non-Maxwellian ion tail and net thermonuclear power can be produced under conditions far less restrictive than Lawson's criterion [4]. The TCT effect is also expected in the ion tail formation by radio frequency (RF) heating, especially by ion cyclotron heating (ICH). Fundamental minority deuteron heating can lead to a two-component velocity distribution suitable for the production of abundant fusion reactions [5-7]. Second harmonic heating of deuterons in a 50:50 D-T plasma can also lead to significant enhancement of the reaction rate [6-9].

The RF-enhancement of high energy tail of the neutral-beam-injected particles was investigated by Kesner [10] and by Pekkari et al. [11]. Calculation was made for a deuteron beam in a triton plasma. It was found that up to 40% of the injected power can be replaced by RF power without serious degrading the fusion output for different mixtures of 150keV deuteron beam and the second harmonic ion cyclotron range of frequency (ICRF) waves [10]. For fundamental heating of deuteron beam ions as a minority ion component, RF-induced velocity diffusion was found to cause significant enhancement of the high energy ion tail and an efficient use of NB and RF power was indicated [11].

In any case, high energy tail ions have significant influence on the reaction rate. Unless plasma energy confinement time is adequately long, achievement of high-Q plasma may be difficult also in the presence of the high energy ion tail, where Q is the fusion power multiplication factor. Even in that case, such an analysis on reactivity enhancement is important in understanding of the non-Maxwellian feature of the ion distribution function. The reactivity enhancement factor defined by the ratio of the enhanced reactivity to Maxwellian one is a measure of deformation of the
distribution from Maxwellian. It is also interesting to examine the
difference between ion tails formed by NBI and ICH and their combined
effects from the reactivity enhancement point of view.

In the present paper, we investigate the fusion reactivity enhancement
due to the deuteron tail formation by NBI and second harmonic ICH with
various deuteron beam energy, power, and RF power in the 50:50 D-T plasma.
The analyses are based on the local Fokker-Planck calculation. Our main
interest is in the deformation in velocity space of the deuteron
distribution function and difference in aspects of the reactivity
enhancement by two schemes of ion tail formation.

In Section 2 we analytically evaluate the ion tail formed by NBI and
ICH. The reactivity enhancement due to the ion tail is examined in
Section 3. Numerical analyses are performed by using a two-dimensional
Fokker-Planck code in Section 4. Section 5 presents the conclusion.

2. EVALUATION OF ION TAIL

2.1 Basic equation

In this section we study the deuteron tail formation by NBI and second
harmonic ICH, by taking notice of the velocity distribution function of
deuterons, \( f = f(v) \), at a local position. The time evolution of \( f \) during NBI
and second harmonic ICH under the influence of Coulomb collisions is
described by

\[
\frac{\partial f}{\partial t} = \left( \frac{\partial f}{\partial t} \right)_{QL} + \left( \frac{\partial f}{\partial t} \right)_{FP} + \left( \frac{\partial f}{\partial t} \right)_{LOSS} + S .
\] (1)

The quasi-linear diffusion term due to ICRF waves, \( \left( \frac{\partial f}{\partial t} \right)_{QL} \), is
described as

\[
\left( \frac{\partial f}{\partial t} \right)_{QL} = \frac{1}{v_L} \frac{\partial}{\partial v_L} \left( v_L D_L \frac{\partial f}{\partial v_L} \right) ,
\] (2)

where RF-induced diffusion in the velocity coordinate parallel to the
distribution from Maxwellian. It is also interesting to examine the
difference between ion tails formed by NBI and ICH and their combined
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In the present paper, we investigate the fusion reactivity enhancement
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\]

The quasi-linear diffusion term due to ICRF waves, \( \left( \frac{\partial f}{\partial t} \right)_{\text{QL}} \), is
described as

\[
\left( \frac{\partial f}{\partial t} \right)_{\text{QL}} = \frac{1}{\nu_L} \frac{\partial}{\partial \nu_L} \left( \nu_L D_{\perp} \frac{\partial f}{\partial \nu_L} \right), \tag{2}
\]

where RF-induced diffusion in the velocity coordinate parallel to the
magnetic field is neglected, since it has a small contribution \((12)\). The perpendicular diffusion coefficient due to the second harmonic ICRF waves, \(D_\perp\), is given by

\[
D_\perp = K_2 \frac{J_1^2(k_\perp \frac{v_\perp}{\omega_{0\perp}})}{\omega_{0\perp}}
\]

(3)

where \(v_\perp\) is the velocity perpendicular to the magnetic field, \(\omega_{0\perp}\) the cyclotron frequency of the deuteron, and \(J_1\) the Bessel function of the first kind of order one. The coefficient, \(K_2\), is proportional to the square of wave amplitude. In the present calculation, however, \(K_2\) is given as an external parameter. The perpendicular wave number of the ICRF wave, \(k_\perp\), is determined from the cold plasma dispersion relation \((13)\),

\[
\frac{c^2 k_\perp^2}{\omega^2} = K_\perp - \frac{K_\perp^2}{K_\parallel^2} \approx 2.4 \frac{\omega_{0\perp}^2}{\omega_{0\perp}^2}
\]

(4)

where \(c\) is the speed of light, \(\omega = 2\omega_{0\perp}\) is the frequency of the RF wave, \(\omega_{0\perp}\) is the deuteron plasma frequency, and \(K_\perp\) and \(K_\parallel\) are the well-known components of the cold plasma dielectric tensor \((14)\). The argument of \(J_1\), which is approximated as \(k_\perp v_\perp/\omega_{0\perp} \approx 6 \times 10^{-8} \sqrt{E_0 n_0/B}\) (\(E_0\) : deuteron energy in keV, \(n_0\) : deuteron density in \(\text{cm}^{-3}\), \(B\) : magnetic field in Tesla), is about unity for \(E_0 = 100\text{keV}, n_0 = 5 \times 10^{13}\text{cm}^{-3}\), and \(B = 5\text{T}\).

The source term, \(S\), originates from NBI,

\[
S = \frac{S_0}{2\pi v_\parallel^2} \delta(v-v_\parallel) p(\theta)
\]

(5)

where \(v\) and \(\theta \equiv \sin^{-1}(v_\perp/v)\) are the particle speed and pitch angle in spherical polar coordinates, respectively. The monochromatic injection energy of \(E_0 = m_0 v_\parallel^2/2\) is assumed. The deposited beam power density is \(P_{\text{NBI}} = E_0 S_0\), and \(p(\theta)\) denotes the distribution function with respect to the injection angle (\(\int p(\theta)\sin\theta d\theta = 1\)). It is assumed that the beam is injected symmetrically with respect to \(v_\parallel\), where \(v_\parallel\) is the velocity.
parallel to the magnetic field.

As for the collision term, \((\partial f/\partial t)_{FP}\), we adopt the linearized Fokker-Planck one.

\[
(\frac{\partial f}{\partial t})_{FP} = \frac{\Gamma_0}{v^2} \left\{ \frac{\partial}{\partial v} \left\{ \alpha_1(v) f + \alpha_2(v) \frac{\partial f}{\partial v} \right\} \\
+ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left\{ \alpha_3(v, \theta) \frac{\partial f}{\partial \theta} \right\} \right\},
\]

(6)

where \(\Gamma_0 = 4\pi z^2 e^4 \ln A / m_0^2\) \((z_0=1)\) is the charge number of the deuteron, \(\ln A\) the Coulomb logarithm, \(e\) the elementary charge, and \(m_0\) the deuteron mass, which are in cgs-esu units. The coefficients, \(\alpha_1(v), \alpha_2(v),\) and \(\alpha_3(v, \theta)\), are the usual linearized collision ones:

\[
\alpha_1(v) = \sum_j \alpha_j(v) = \sum_j (\frac{Z_j}{z_0})^2 \frac{m_j}{m_0} n_j \\
\times \left\{ - \frac{v}{\nu_{th,j}} \left(\frac{2}{\pi}\right)^{1/2} \exp\left(-\frac{v^2}{2\nu_{th,j}}\right) + \phi\left(\frac{v}{\sqrt{2} \nu_{th,j}}\right) \right\},
\]

(7)

\[
\alpha_2(v) = \sum_j \frac{\nu_{th,j}}{v} \frac{m_j}{m_0} \alpha_j(v),
\]

(8)

\[
\alpha_3(v, \theta) = \sum_j (\frac{Z_j}{z_0})^2 \frac{n_j \sin \theta}{2v} \left\{ \frac{\nu_{th,j}}{v} \left(\frac{2}{\pi}\right)^{1/2} \exp\left(-\frac{v^2}{2\nu_{th,j}}\right) \right\} \\
+ \left(1 - \frac{\nu_{th,j}^2}{v^2}\right) \phi\left(\frac{v}{\sqrt{2} \nu_{th,j}}\right),
\]

(9)

where \(z_j, m_j, n_j, \nu_{th,j} = \sqrt{T_j/m_j}\), and \(T_j\) are the charge number, mass, number density, thermal speed, and temperature of species \(j\), respectively, and the error function, \(\phi(x)\), is defined as \(\phi(x) = (2/\sqrt{\pi}) \int_0^x \exp(-y^2) dy\).

The loss term, \((\partial f/\partial t)_{LOSS}\), corresponds to the energy conduction loss described as the following simple form:

\[
(\frac{\partial f}{\partial t})_{LOSS} = \frac{1}{v^2} \frac{\partial}{\partial v} \left( \frac{v^2}{2\tau_e(v)} f \right),
\]

(10)
The macroscopic energy confinement time of deuterons given by 
\[ \tau_E = \int d\tilde{u} \left\{ u^2 f(\tilde{u}) \right\} / \int d\tilde{u} \left\{ u^2 f(\tilde{u})/\tau_E(\tilde{u}) \right\} \] becomes equal to \( \tau_E \) when \( \tau_E(\tilde{u}) \) is constant. The particle loss is assumed to occur in the very low energy region.

2.b Analytical solution

The distribution function, \( f(u, \theta) \), is expanded by using Legendre functions of the first kind, \( P_n(\xi) \), as

\[ f(u, \xi) = F_0(u) + F_2(u)P_2(\xi) + F_4(u)P_4(\xi) + \cdots, \quad \text{(11)} \]

where \( \xi = \cos \theta \). The zero-order coefficient, \( F_0(u) \), corresponds to the angle-averaged distribution function:

\[ F_0(u) = \frac{1}{2} \int_{-1}^{1} d\xi \int_{0}^{\alpha} f(u, \xi). \quad \text{(12)} \]

The quasi-linear diffusion term due to ICRF waves given by Eq.(2) is rewritten in spherical polar coordinates,

\[ \left( \frac{\partial f}{\partial t} \right)_{\alpha} = \frac{1}{u^2} \frac{\partial}{\partial u} \left\{ u^2 (1-\xi^2) \frac{\partial f}{\partial u} - u \xi (1-\xi^2) \frac{\partial f}{\partial \xi} \right\} 
- \frac{1}{u^2} \frac{\partial}{\partial \xi} \left\{ u \xi (1-\xi^2) \frac{\partial f}{\partial u} - \xi^2 (1-\xi^2) \frac{\partial f}{\partial \xi} \right\}. \quad \text{(13)} \]

By averaging Eq.(1) in the steady state over \( \xi = \cos \theta \) and truncating the Legendre expansion of \( f \) at zero-order, we obtain

\[ 0 = \frac{K_0}{\nu^2} \frac{d}{d\nu} \left\{ \nu^2 C(\nu) \frac{dF_0}{d\nu} \right\} - \frac{G_0}{\nu^2} \frac{d}{d\nu} \left\{ \alpha_1(\nu)F_0 + \alpha_2(\nu) \frac{dF_0}{d\nu} \right\} 
+ \frac{1}{\nu^2} \frac{d}{d\nu} \left\{ \frac{\nu^3}{2\pi \bar{\nu}(\nu)} F_0 \right\} - \frac{S_0}{4\pi \nu^2} \frac{d}{d\nu} \left\{ U(\nu_0 - \nu) \right\}. \quad \text{(14)} \]
where $F_0(v)$ is simplified to $F_0$, $U(v_0 - v)$ denotes the step function, and

$$C(v) = \int_0^1 d\xi \frac{1}{1 - \xi^2} J_0^2 \left( \frac{k_1 v}{\omega c_0} \sqrt{1 - \xi^2} \right).$$

(15)

This equation easily reduced to the equation,

$$\frac{dF_0(v)}{dv} = A(v)F_0(v) = B(v)U(v_0 - v),$$

(16)

where

$$A(v) = \frac{\Gamma_0 \alpha_1(v) + \frac{\nu^3}{2\pi} \zeta'(v)}{\Gamma_0 \alpha_2(v) + K_1 \nu^2 C(v)},$$

(17)

$$B(v) = \frac{S_0}{4\pi} \frac{1}{\Gamma_0 \alpha_2(v) + K_1 \nu^2 C(v)}. $$

(18)

Equation (16) is the fundamental equation for the analyses in Section 3. In the region of our interest, $v_{th,D} \ll v \ll v_{th,e}$, $A(v)$ and $B(v)$ can be reduced to

$$A(v) \sim \frac{\frac{\nu^3}{2} \epsilon^3 + \frac{\nu^3}{2} \zeta'(v)}{T_e \nu^2 / m_0 + T_i \nu^2 / m_0 + K_2 \nu^2 C(v)},$$

(19)

$$B(v) \sim \frac{S_0 \zeta'(v)}{4\pi} \frac{1}{T_e \nu^2 / m_0 + T_i \nu^2 / m_0 + K_2 \nu^2 C(v)},$$

(20)

where $T_i = T_D = T_i$ is assumed, and the critical speed, $v_c$, and the slowing-down time, $\tau_s$, are given by

$$v_c = (\frac{3\sqrt{\pi}}{4} \frac{m_e}{m_0} z_1)^{1/3} \sqrt{2} \ v_{th,e} \ \text{with} \ \ z_1 = \sum_{j=0}^{n_e} \frac{\zeta(j) z_1^2}{n_e m_j}.$$ 

(21)

$$\tau_s = \frac{m_0^2}{2\pi z_1^2 \epsilon n_e} \ln \left( \frac{\nu_c^3}{2z_1} \right).$$

(22)
In the 50:50 D-T plasma, the critical deuteron energy, \( E_c = \frac{m_d T_e^2}{2} \), is about 16.5\( T_e \), and the value of \( \tau_s \) is given as \( \tau_s = 4 \times 10^{13} T_e^{3/2}/n_e \ln \Lambda \) (\( \tau_s \) is in sec, \( T_e \) in keV, and \( n_e \) in cm\(^{-3} \)). Under the approximation, \( J_f^2 (k \perp v_{\perp} / \omega_{ci}) \sim (k \perp v_{\perp} / 2 \omega_{ci})^2 \), \( C(v) \) can be also reduced to

\[
C(v) \sim \frac{2}{15} \frac{k^2 v^2}{\omega_{ci}}.
\]  

(23)

The solution to Eq.(16) is formally expressed as

\[
F_0(v) = \exp(-\int_0^v dv' A(v'))
\]

\[
\times \left\{ F_0(0) + \int_0^v dv' B(v') \exp(\int_0^{v'} dv'' A(v'')) \right\},
\]

\( F_0(v) = \exp(-\int_0^v dv' A(v')) \)

\[
\times \left\{ F_0(0) + \int_0^{v'} dv'' B(v') \exp(\int_0^{v''} dv''' A(v''')) \right\},
\]

(24a)

(24b)

where the value of \( F_0(0) \) is determined from the constraint of particle conservation. To obtain \( F_0(v) \), the numerical integration of Eq.(16) from \( v=0 \) is much easy compared with the integration in Eq.(24), though a small number of iterations are necessary for the particle conservation. The profile of the angle-averaged distribution function, \( F_0(v) \), obtained from Eq.(16) is shown in Figs.4 and 6 in the next section. For fundamental minority heating of the injected beams, the corresponding solution has been obtained in Ref. \([11] \).

Equation (16) is the exact equation for \( F_0(v) \) when RF wave is absent: the linearized Fokker-Planck term, energy loss term, and the source term due to NBI are exactly reduced to the corresponding terms in Eq.(14) by integration over \( \xi \). The higher order terms, \( F_{2N} (N=1,2, \ldots) \), which originate from the average of Eq.(13), are neglected in Eq.(14). The higher order term in the Legendre expansion due to RF waves is discussed in
Section 4 on numerical analysis.

The deposited RF power density, \( P_{RF} \), is determined by the whole profile of \( f(u) \), while \( P_{NBI} \) is determined by the source term. The deposited RF power density calculated from the zero-order analysis is given by

\[
P_{RF} = \int d^3 u \frac{1}{2} m_0 \nu^2 \left( \frac{\partial f}{\partial t} \right) u l
\]

\[
= 4\pi \int_0^\infty d\nu \left\{ \nu^2 C(\nu) \frac{dF_0}{d\nu} \right\}
\]

\[
= -4\pi m_0 k_2 \int_0^\infty d\nu \nu^2 C(\nu) \frac{dF_0}{d\nu}
\]

\[
= 4\pi m_0 k_2 \int_0^\infty d\nu \nu^2 C(\nu) \{ A(\nu)F_0(\nu) - B(\nu)\tilde{F}(\nu_0 - \nu) \}.
\]

where use has been made of Eq.(16). If the distribution function, \( F_0(\nu) \), is Maxwellian with the temperature, \( T_0 \), the deposited power density is easily obtained as

\[
<P_{RF}>_M = 2k_2 \frac{\nu_0 T_0 k_2^2}{\omega_0^2}.
\]

when \( k_\perp \rho_{th} = k_\perp v_{th}/\omega_0 \ll 1 \) and \( J_2^2 \sim (k_\perp v_{\perp}/2\omega_0)^2 \) ( \( v_{th} = \sqrt{T_0/m_0} \) is the deuteron thermal speed ). We use, hereafter, the normalized quantities, \( \tilde{k}_2 \equiv k_2 \tau_s k_2^2/\omega_0^2 \) and \( \tilde{\rho} \equiv \rho_{\perp}/\nu_0 T_0 \) ( \( \tau_s \) is the slowing-down time given by Eq.(22) ). Equation (26) is rewritten as \( <P_{RF}>_M = 2\tilde{k}_2 \). The normalized power density, \( \tilde{\rho} \propto PT_0^{1/2}/\nu_0 n_0 T_0 \), is about 6.25 for \( P = 1W/cm^3 \), \( T_0 = 0.5keV \), and \( n_0 = n_e/2 = 5 \times 10^{13}cm^{-3} \). When the deformation of the distribution function due to the RF wave is small, the deposited RF power density, \( P_{RF} \), is linearly proportional to \( \tilde{k}_2 \). This linear relation breaks for large \( \tilde{k}_2 \), since the profile of \( F_0(\nu) \) depends on the value of \( \tilde{k}_2 \). Figure 1 shows the relation between \( \tilde{k}_2 \) and \( P_{RF} \) obtained from Eqs.(16) and (25) for cases of ICH ( triangle ) and NBI + ICH ( circle ). The dashed line in the figure denotes \( <P_{RF}>_M \) given by Eq.(26). The NBI power density is fixed as \( \tilde{P}_{NBI} = \)}
\[ \begin{aligned} &2 \; \text{with} \; E_0/T = 20, \, \text{where the temperature of each component is the same,} \; T = T_e = T_B = T_\gamma. \; \text{The energy confinement time, } \tau_E(\nu), \, \text{is assumed to be} \\
&\text{infinitely large. The wave number of the RF wave, } k_\perp, \, \text{is chosen as} \\
k_\perp v_0/\omega_{pe} = 0.9 \; \left( v_0 = \sqrt{2E_0/m_0} \right). \, \text{The deposited RF power density becomes} \\
large \text{for the combination of NBI and ICH, since RF diffusion becomes large} \\
in the higher energy region. \, \text{In contrast with second harmonic cyclotron heating, the} \\
\text{corresponding linear relation correctly holds for fundamental} \\
cyclotron \text{heating of minority ions even when } F_0 \text{ is deformed by RF waves} \\
[5]. \\
\end{aligned} \]

3. REACTIVITY ENHANCEMENT DUE TO ION TAIL

3.a Fusion reactivity

The reactivity, \( \langle \sigma v \rangle \), is represented by

\[ \langle \sigma v \rangle = \int d\nu \, d\omega \, f_D(\nu) \, f_T(\omega) \, \sigma(u) \, u / n_0 n_T, \]  \hfill (27) \]

where \( \sigma(u) \) is the fusion cross section, \( u \) is the relative speed, \( u = |\vec{v} - \vec{\omega}| \), \( f_D \) \( ( f_T ) \) \( ( \nu_0 \) \( ( n_T ) \) are the distribution function and the particle number density of deuterons \( ( \text{tritons} \) \), respectively. The relative speed, \( u \), is expressed by using the angle, \( \beta \), between \( \vec{v} \) and \( \vec{\omega} \):

\[ u^2 = v^2 - 2vw \cos \beta + w^2, \]  \hfill (28) \]

where \( v = |\vec{v}| \) and \( w = |\vec{\omega}| \). By using the relation, \( u \, du = vw \sin \beta \, d\beta \) and taking \( f_T(\omega) \) to be isotropic, i.e., \( f_T(\omega) = f_T(w) \), we obtain the '\( \sigma v \)-function' depending only on \( u \),

\[ G(\nu) \equiv \int d\omega \, f_T(\omega) \, \sigma(u) \, u \]

\[ = 2\pi \int_0^\infty dw \, w^2 \, f_T(\omega) \int_0^{\pi} d\beta \sin \beta \, \sigma(u) \, u \]
2 with $E_0, T = 20$. where the temperature of each component is the same, $T = T_e = T_D = T_T$. The energy confinement time, $\tau_e(u)$, is assumed to be infinitely large. The wave number of the RF wave, $k_\perp$, is chosen as $k_\perp \nu_0/\omega_{ci} = 0.9$ ($\nu_0 = \sqrt{2E_0/m_0}$). The deposited RF power density becomes large for the combination of NBI and ICH, since RF diffusion becomes large in the higher energy region. In contrast with second harmonic cyclotron heating, the corresponding linear relation correctly holds for fundamental cyclotron heating of minority ions even when $F_0$ is deformed by RF waves [5].

3. REACTIVITY ENHANCEMENT DUE TO ION TAIL

3.a Fusion reactivity

The reactivity, $\langle \sigma u \rangle$, is represented by

$$\langle \sigma u \rangle = \int d\tilde{u} \, d\tilde{w} \, f_0(\tilde{u}) \, f_T(\tilde{w}) \, \sigma(u) \, u / n_0 n_T.$$  \hspace{1cm} (27)

where $\sigma(u)$ is the fusion cross section, $u$ is the relative speed, $u = |\tilde{u} - \tilde{w}|$, $f_0$ ($f_T$) and $n_0$ ($n_T$) are the distribution function and the particle number density of deuterons (tritons), respectively. The relative speed, $u$, is expressed by using the angle, $\beta$, between $\tilde{u}$ and $\tilde{w}$:

$$u^2 = v^2 - 2uw \cos \beta + w^2.$$  \hspace{1cm} (28)

where $v = |\tilde{u}|$ and $w = |\tilde{w}|$. By using the relation, $u \, du = uv \sin \beta \, d\beta$ and taking $f_T(\tilde{w})$ to be isotropic, i.e., $f_T(\tilde{w}) = f_T(w)$, we obtain the $\langle \sigma u \rangle$-function depending only on $v$.

$$G(v) \equiv \int d\tilde{w} \, f_T(\tilde{w}) \, \sigma(u) \, u$$

$$= 2\pi \int_0^\infty dw \, w^2 \, f_T(w) \int_0^\pi d\beta \, \sin \beta \, \sigma(u) \, u$$
\[
\begin{align*}
    &= 2\pi \int_0^{\infty} dw \, w^2 \, f_T(w) \int_{|v-w|}^{v+u} du \, \frac{u^2}{iw} \, \sigma(u) \\
    &= \frac{2\pi}{v} \int_0^{\infty} dw \, w \, f_T(w) \int_{|v-w|}^{v+u} du \, u^2 \, \sigma(u).
\end{align*}
\]

The reactivity, \( \langle \sigma v \rangle \), is calculated by using this '\( \sigma \)T-function', \( G(v) \);

\[
\begin{align*}
    \langle \sigma v \rangle &= \frac{1}{n_0 n_T} \int d\vec{v} \, f_0(\vec{u}) \, G(v) \\
    &= \frac{2\pi}{n_0 n_T} \int_0^{\infty} dv \, v^2 \int_0^{\pi} d\theta \, \sin\theta \, f_0(\vec{v}) \, G(v) \\
    &= \frac{4\pi}{n_0 n_T} \int_0^{\infty} dv \, v^2 \, F_0(v) \, G(v).
\end{align*}
\]

Note that only the zero-order part in the Legendre expansion, \( F_0(v) \), is needed in calculating \( \langle \sigma v \rangle \), when \( f_T(\vec{w}) \) is isotropic. In order to enhance the reactivity, the particle number in the high energy tail must be effectively raised; the tail near the peak point of \( G(v) \) largely contributes to the reactivity enhancement. The area enclosed by \( v^2 F_0(v) G(v) \) and \( v \)-axis gives the reactivity. The aspects of the reactivity enhancement can be understood by examining the effect of the ion tail on \( v^2 F_0(v) G(v) \).

At first we calculate the reactivity, \( \langle \sigma v \rangle \), for the Maxwellian D-T plasma with the same temperature, \( T_i = T_D = T_T \). Hereafter we simply replace \( \langle \sigma v \rangle \) for the Maxwellian D-T plasma by \( \langle \sigma v \rangle_0 \). The fitting function for D-T fusion cross section, \( \sigma(u) \), given in Ref. [15] is used through the present work. The value of the reactivity calculated from Eq. (30) agrees with that cited in Ref. [15]. Figure 2 shows (A) \( G(v) \) and (B) \( v^2 F_0(v) G(v) \) for the Maxwellian D-T plasma with the temperature of (i) 5keV and (ii) 10keV. The abscissa in the figure indicates the normalized deuteron speed, \( v = u / u_{th} \) ( \( u_{th} = \sqrt{T_i/n_0} \) is the deuteron thermal speed for each temperature ). The quantities in the ordinates are normalized as follows: \( F_0(v) = F_0(v) u_{th}^3/n_0 \) with non-dimension and \( G(v) = G(v) / n_T u_{th} \) in unit of \( cm^2 \). The normalized distribution function, \( F_0(v) \), of Maxwellian deuterons.
in Fig.2 is \( F_0(v) = (2\pi)^{-\frac{3}{2}} \exp(-v^2/2) \). The \( \sigma v \)-function \( G(v) \) can be approximated as \( G(v) \sim n_T \sigma(v) v \) for \( v \gg w_{th} \) ( \( w_{th} \) is the triton thermal speed), and the peak position of \( G(v) \) is almost the same as that of \( \sigma(v) v \). The maximum value of \( \sigma v \) for D-T reaction is \( (\sigma v)_{\text{max}} = 1.67 \times 10^{-13} \text{cm}^3/\text{sec} \) at \( v = v_* \) (the deuteron energy, \( E_0 = m_0 \tilde{E}_0/2 \), is 127 keV). Therefore, this peak point is closer to the bulk for \( T_i = 10 \text{keV} \) than to that for \( T_i = 5 \text{keV} \), and \( \langle \sigma v \rangle_H = 1.09 \times 10^{-16} \text{cm}^3/\text{sec} \) for \( T_i = 10 \text{keV} \) is much larger than \( \langle \sigma v \rangle_H = 1.29 \times 10^{-17} \text{cm}^3/\text{sec} \) for \( T_i = 5 \text{keV} \).

3.b Beam- and RF-induced enhancement of reactivity

For the case of only NBI heating, the reactivity can be simply expressed as follows. The distribution function of deuterons is assumed to consist of the Maxwellian bulk and the tail component described as

\[
F_{0,\text{tail}}(v) = \frac{S_0 \tau_\varepsilon}{4\pi h} \frac{1}{v^3 + v_c^3/h},
\]

where \( \tau_\varepsilon \) and \( v_c \) are functions of \( T_e \). \( \tau_\varepsilon \) is assumed to be constant, and \( h = 1 + \tau_\varepsilon/2\tau_\varepsilon \). The distribution of tritons is Maxwellian with the temperature, \( T_i \), which is the same as that of bulk deuterons. The reactivity is then obtained as

\[
\langle \sigma v \rangle = \langle \sigma v \rangle_H \left\{ 1 - \frac{P_{\text{NBI}} \tau_\varepsilon}{n_T E_0 h} \frac{1}{3} \ln\left( \frac{h v_*^3}{v_c^3} + 1 \right) \right\} + \frac{P_{\text{NBI}} \tau_\varepsilon}{n_T E_0 h} \frac{\langle \sigma v \rangle_{\text{max}}}{\langle \sigma v \rangle_H} \int_{0}^{v_*} \frac{v^2 G(x)}{v^3 + v_c^3/h} dv,
\]

where \( G(x) = G(v)/n_T \langle \sigma v \rangle_{\text{max}} \) and \( x = v/v_* \). In order to evaluate the reactivity enhanced by the deuteron tail, we propose a simple function for \( G(v) = n_T \langle \sigma v \rangle_{\text{max}} G(x) \),

\[
G(x) = G_k(x) + \hat{G}_k(x - 0.97) \quad \text{for} \quad x_k < x < x_{k+1} \quad (k = 1.2.3),
\]
where \( x \) is the normalized speed, \( x = \frac{v}{u_e}, x_1 = 0.97 - \hat{\gamma}_1 \hat{\gamma}_1, x_2 = 0.97, x_3 = 0.97 - (\hat{\gamma}_2 - \hat{\gamma}_3) / (\hat{\gamma}_2 - \hat{\gamma}_3), \) and \( x_4 = 3. \) The function \( \hat{\gamma}(x) \) is zero for \( x < x_1. \) Coefficients are related with the temperature: \( \hat{\gamma}_1 = \hat{\gamma}_2 = 1.06 - 0.053 \sqrt{T_1}, \hat{\gamma}_3 = 0.33, \hat{\gamma}_1 = 1/((0.40 - 0.037 T_1), \hat{\gamma}_2 = -1/(0.91 - 0.16 \sqrt{T_1}), \) and \( \hat{\gamma}_3 = -0.11, \) where \( T_1 \) is in unit of keV. The distribution function of tritons is assumed to be Maxwellian with the temperature, \( T_1, \) which is much smaller than \( E. (T_1 < 30 \text{keV}.) \) This approximate function is useful when the tail spreads beyond the cutoff speed, \( x_1 u_e. \) The cutoff deuteron energy, \( E_1 = \frac{m_0 c^2 v^2}{2}, \) is \( E_1 = 38 \text{keV} \) for \( T_1 = 0 \text{keV}, E_1 = 26 \text{keV} \) for \( T_1 = 5 \text{keV}, \) and \( E_1 = 15 \text{keV} \) for \( T_1 = 10 \text{keV}. \) By using an approximate function for \( \hat{\gamma}(x) \) given by Eq. \((33), \) we can easily integrate Eq. \((32), \)

\[
\int_{0}^{x_0} \frac{v^2 \hat{\gamma}(x)}{v^2 + \hat{\gamma}_3 / \hat{\gamma}_1} dx = \int_{0}^{x_0} \frac{x^2 (\hat{\gamma}_1 - 0.97 \hat{\gamma}_2)}{x^2 + 1} dx

\[
= \left[ \frac{\hat{\gamma} \hat{\gamma}_1 (x^2 - 1)}{\hat{\gamma} \hat{\gamma}_1 (x^2 - 1)} + \frac{\hat{\gamma}}{\hat{\gamma}} \ln \left( \frac{x^2 - 1}{x^2 - 1} \right) \right]_{x_0}^{x_0}, \tag{34}
\]

where \( x = h^{1/3} / v_e = x h^{1/3} / v_e, x_0 = h^{1/3} / v_e, \) and \( \gamma = v_e / c h^{1/3}. \) The dependence of the reactivity on the temperature is shown in Fig. 3, where the temperature of each component is the same. \( T = T_c = T_1 \), the beam power density is chosen constant as \( \hat{\rho}_{\text{BE}} = 10 \text{ at } T = 5 \text{keV}, \) and the injection beam energy, \( E_0, \) is (a) 50 keV, (b) 100 keV, and (c) 200 keV. The energy confinement time is taken to be infinitely large. The dashed line denotes \( \angle \hat{\rho}_{\text{BE}} \angle, \) the solid line for \( \angle \hat{\rho}_{\text{BE}} \angle \) is calculated from Eqs. \((16)\) and \((30), \) and the black circles are obtained from Eqs. \((32)\) and \((34), \) which agree well with the solid line. The error in the results by Eqs. \((32)\) and \((34)\) comes from the approximation of \( F_0, \text{tail} \langle v \rangle, \) Eq. \((31), \) and of \( \hat{\gamma}(x), \) Eq. \((33), \) and it becomes large for lower injection beam energy. When \( E_0 \) is close to \( T_1, \) e.g., the case of \( E_0 = 50 \text{keV} \) and \( T_1 = 25 \text{keV}, \) the profile of the tail component is much different from Eq. \((31), \) and when \( E_0 \) approaches the cutoff energy, \( E_1 = \frac{m_0 c^2 v^2}{2}, \) e.g., the case of \( E_0 = 50 \text{keV} \) and \( E_1 = 34 \text{keV} \) for \( T_1 = 2 \text{keV}, \) Eq. \((33)\) for \( \hat{\gamma}(x) \) is insufficient to integrate Eq. \((32). \)

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Now we examine the reactivity enhancement for ICRF heating as well as NBI heating by using Eq. (16) for $F_0(v)$. To measure the enhancement, we introduce the reactivity enhancement factor, $\eta \equiv \langle \alpha_1 \rangle / \langle \alpha_0 \rangle$. In terms of the increase of $\eta$, the ion tail formation for $T_i = 5$keV is more effective than that for $T_i = 10$keV, because the higher speed particles in the Maxwellian distribution are more accessible to $G(v)$ near its peak point, which results in the sufficiently large Maxwellian reactivity for $T_i = 10$ keV. Therefore we study the reactivity enhancement mainly for the case of $T_i = 5$keV. The bulk temperature of each component is assumed to be the same, $T = T_e = T_0 = T_i$, and distribution functions of electrons and tritons are Maxwellian. The energy confinement time, $\tau_E$, is infinitely large. We choose values of plasma parameters so that $k_{L0}/\omega_{ce}$ does not much exceed unity, e.g., $k_{L0}/\omega_{ce} \sim 1$ for $E_0 = 100$keV, $n_0 = 5 \times 10^{13} \text{cm}^{-3}$, and $B = 5T$.

Figure 4 shows the formation of the ion tail for only NBI with (a) $E_0 = 50$keV, (b) 100keV, (c) 200keV, and for (d) only RF heating. The heating power density is $\bar{P}_{\text{NBI}} = 2$ (dashed line) and $\bar{P}_{\text{NBI}} = 5$ (solid line) for the cases (a-c), and $\bar{P}_{\text{RF}} = 2.1$ ($\bar{P}_{\text{RF}} = 2$, $\bar{k}_2 = 1$ : dashed line) and $\bar{P}_{\text{RF}} = 5.0$ ($\bar{P}_{\text{RF}} = 4.1$, $\bar{k}_2 = 2.05$ : solid line) for the case (d). The angle-averaged distribution function, $F_0(v)$, given by Eq. (16) is shown in Fig.4A in the logarithmic scale, and $v^2F_0(v)G(v)$ is in Fig.4B. The $\alpha_0$-function, $G(v)$, is the same as that shown in Fig.2A-(i) for $T = 5$keV.

As seen in Figs.4B-(a), (b), and (c), the beam-induced tail typically presents two humps in $v^2F_0(v)G(v)$. The lower energy hump comes from the bulk component which has almost the same profile as shown in Fig.2B-(i), and the higher energy one is due to the tail component which is heightened with the NBI power density. For higher energy beam injection, $E_0 = 100$keV and 200keV, the beam-induced tail is placed in the energy region near the peak point of $G$. The hump in $v^2F_0G$ becomes much larger than that for $E_0 = 50$keV, and the reactivity is effectively enhanced by the ion tail. On the other hand, a high-v 'wing' rather than a hump is formed in the case of only RF heating, as shown in Fig.4B-(d). This 'wing' spreads to the high energy region with the increase of the RF power density. The effect of RF heating on the reactivity enhancement is weak compared with the case of NBI heating with $E_0 = 100$ keV and 200keV.
The dependence of $\eta$ on the deposited power density, $\dot{P}_d = \dot{P}_{NB1} + \dot{P}_{RF}$ is shown in Fig.5. The plasma temperature is (i) $T = 5\,keV$ and (ii) $10\,keV$. The beam injection energy is chosen as (a) $E_0 = 50\,keV$ (cross), (b) $100\,keV$ (circle), and (c) $200\,keV$ (square). For the case of only NBI heating (solid lines for (a), (b), and (c)), $\eta$ is linearly proportional to $\dot{P}_{NB1}$ as is given by Eq.(32). On the other hand, for (d) only RF heating (triangle), $\eta$ does not increase appreciably in the low power range, because the tail is not spread by RF diffusion to the large $G(v)$ region. When the RF power becomes large, $\eta$ becomes larger than that for the case of (a) $E_0 = 50\,keV$, and the derivative, $d\eta/d\dot{P}_d$, becomes as large as that for the case of (b) $E_0 = 100\,keV$.

Figure 6 shows the effect of the combination of NBI and RF heating. The deposited beam power density is $\dot{P}_{NB1} = 2$ with the beam energy, $E_0$, of (a) $50\,keV$, (b) $100\,keV$, and (c) $200\,keV$. The RF parameter is chosen as, $\tilde{K}_2 = 1$, and the following deposited RF power density, $\dot{P}_{RF}$, is obtained for each case: (a) 2.3, (b) 2.6, and (c) 2.8. In Fig.6 we show the profiles of (A) $F_0(v)$ and (B) $v^2F_0(v)G(v)$. Each beam-induced tail is enhanced owing to RF-induced velocity diffusion, and a 'wing' in $v^2F_0(v)G(v)$ is added to each hump due to only NBI heating. When the beam injection energy, $E_0$, is lower than $E_*$ at which $G$ becomes maximum, this addition of the 'wing' is preferable for the reactivity enhancement in comparison with the case of $E_0 > E_*$. The value of $\eta$ is increased by RF heating from 2.5 ($\dot{P}_{NB1} = 2$) to 3.6 by adding $\dot{P}_{RF} = 2.6$ for $E_0 = 100\,keV$, and from 3.1 ($\dot{P}_{NB1} = 2$) to 4.0 by adding $\dot{P}_{RF} = 2.8$ for $E_0 = 200\,keV$. The dependence of $\eta$ on $\dot{P}_d$ for the combined heating is also shown in Fig.5 by dashed lines for cases (b) $E_0 = 100\,keV$ (circle) and (c) $200\,keV$ (square). The value of $\dot{P}_d$ is increased by adding $\dot{P}_{RF}$ to the fixed value of $\dot{P}_{NB1} = 6$ for $T = 5\,keV$ and $\dot{P}_{NB1} = 8.3$ for $T = 10\,keV$. For $E_0 = 200\,keV > E_*$, RF diffusion occurs mainly in the region of decreasing $G(v)$ and the derivative, $d\eta/d\dot{P}_{RF}$, is smaller than $d\eta/d\dot{P}_{NB1}$. On the other hand, $d\eta/d\dot{P}_{RF}$ is comparable to $d\eta/d\dot{P}_{NB1}$ for $E_0 = 100\,keV < E_*$, and it becomes larger than $d\eta/d\dot{P}_{NB1}$ when $E_0$ is much smaller than $E_*$. 

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4. NUMERICAL ANALYSIS

In Section 3, we evaluated the reactivity enhancement by using the solution to Eq. (16) which contains the error for the effect of RF heating, and in this section we study the ion tail formation by using a two-dimensional linearized Fokker-Planck code. RF-enhancement of the beam-induced tail in the cases of parallel (co- and counter-) perpendicularly, and nearly-isotropic injection is examined. Also discussed is the dependence of the reactivity enhancement factor on the deposited NB and RF power density. Stationary results are found from the time dependent calculation. In the present calculation, the temperature of each component, T, is 5keV, the value of k_L U_0/ω_eB does not much exceed unity, and the energy confinement time is chosen infinity. The particle loss is determined to satisfy particle conservation, and is assumed to exist only in the very low energy region.

Figure 7A shows F_0(v) and Fig. 7B shows v^2F_0(v)G(v) for (a) the case of only 100keV beam injection with \tilde{\alpha}_{NB} = 2, (b) the combination of parallel injection of 100keV beam and RF heating with \tilde{k}_\perp = 1, (c) perpendicular beam injection and RF heating, and (d) nearly-isotropic beam injection and RF heating. Good agreement is seen between analytical and numerical results for only NBI cases (Fig.4-(b) (dashed line) and Fig.7-(a)). The ion tail in F_0(v) induced by the parallel injected beam is not almost enhanced by RF-induced diffusion (Fig.7A-(b)). On the other hand, in the case of perpendicular injection the beam-induced tail and the reactivity are effectively enhanced by RF-induced diffusion (Fig.7-(c)). The aspects of RF-enhancement of the beam-induced tail in the case of nearly-isotropic injection (Fig.7-(d)) are most similar to those in one-dimensional analysis (Fig.6-(b)).

In the analytic estimation, the higher order coefficients in the Legendre expansion are fully neglected. We investigate here the higher order terms. Figure 8 shows (A) F_0(v), (B) v^2F_0(v)G(v), and (C) the coefficients in the Legendre expansion of f(u,ξ). F_0 (solid line), F_2 (dashed line), and F_4 (dash-dotted line), for only RF heating with \tilde{\alpha}_{RF} = 5. The negative F_2(v) indicates the perpendicular tail. The ratio of
$F_2(v)$ to $F_0(v)$ for $v > 5$ is given by $\frac{F_2(v)}{F_0(v)} = 1.3 \sim 2.3$, which is larger than the value, $\frac{F_2(v)}{F_0(v)} = 10/7$, obtained analytically by Anderson et al. (12) in the high energy limit. Figure 9 shows $\bar{\rho}_{RF}$ versus $\bar{K}_2$ for only RF heating (triangle) and for the combination of 100keV beam injection with $\bar{\rho}_{NB} = 2$ and RF heating. The cases of parallel (cross), perpendicular (square), and nearly-isotropic (circle) injections are indicated. The dashed line in the figure is $\langle \bar{\rho}_{RF} \rangle_M$ which is given by Eq.(26). Deviation of $\bar{\rho}_{RF}$ from $\langle \bar{\rho}_{RF} \rangle_M$ increases with $\bar{K}_2$, though it is negligibly small for only RF heating in the range of $\bar{K}_2 < 1$. Nonlinearity in the relation between $\bar{K}_2$ and $\bar{\rho}_{RF}$ is slightly enhanced as compared with that for 1D analysis, which is shown in Fig.1. The deposited RF power density in the presence of the beam-induced tail is larger than that in the absence of it; coupling of RF waves with the ion can be improved by the beam-induced tail (16). Especially the ion tail induced by the perpendicularly injected beam couples the most with the RF waves.

Figure 10 shows the reactivity enhancement factor, $\eta$, versus the deposited power density, $\bar{\rho}_d = \bar{\rho}_{NB} + \bar{\rho}_{RF}$, for various combinations of the beam and RF parameters for $T = 5$keV. In the figure only NBI cases are shown by solid lines. The injection energy is chosen as (a) $E_0 = 50$keV (cross), (b) 100keV (circle), and (c) 200keV (square). Only RF case (d) is shown by dash-dotted line with triangles. Dashed lines correspond to the cases of combination of parallel beam injection and RF heating, and dotted lines to the cases of combination of perpendicular beam injection and RF heating. There is the threshold power density of $\bar{\rho}_{RF} = 1$ for the tail formation only by RF heating from the viewpoint of the reactivity enhancement. The reactivity enhancement factor for only RF heating with higher power injection exceeds that for only 50keV beam injection. In the cases of only NBI the reactivity increases linearly with the beam power density (solid lines). The rate of increase is dependent on the injection energy. The increase in the power density of 50keV beam can not effectively enhance the reactivity, which is also seen in the contribution of the 50keV-beam-induced tail to $v^2F_0G$ (Fig.4B-(a)). On the other hand, injection of 100keV and 200keV beams can enhance the reactivity more effectively: For $T = 5$keV the reactivity can be enhanced by an order of
magnitude, as compared with $\langle \sigma v \rangle_M$. The rate of increase of the reactivity with the deposited power density, $\dot{P}_d$, for only NBI is deteriorated by RF injection in the combination of parallel injection of high energy beam and RF heating (dashed lines). For $E_0 = 200$ keV, even the combination of the perpendicularly injected beam and RF is not advantageous, and the increase in the beam power density is preferable. This is because RF-enhancement of the beam-induced tail in the higher velocity region beyond the peak point of $G(v)$ does not work very effectively in the reactivity enhancement. On the contrary, the reactivity enhanced by the perpendicularly injected 100keV beam can be further improved by RF injection. In this case RF-induced velocity diffusion works effectively by producing the higher energy tail which is given access to the $'\sigma v'$-function' $G(v)$ in the region closer to its peak point.

The improvement in the reactivity by the RF wave is realized when the ion tail is formed by the perpendicularly injected 100keV beam. The reactivity enhancement factor, $\eta$, is shown in Fig.11 as a function of $P_{NB}/P_d$ for the combination of parallel (cross) or perpendicular (square) injection of 100keV beam and RF heating with $\dot{P}_d = 10$ for $T = 5$keV. Note that the optimum fraction of power density supplied by the perpendicular beam exists; $P_{NB}/P_d \sim 0.7$ for this case. The value of $\eta$ at $P_{NB}/P_d = 1$ and the optimum fraction become small with the decrease of $E_0$. For $E_0 = 50$ keV $\eta$ decreases with the increase of $P_{NB}/P_d$. On the other hand, the reactivity is degraded by RF injection in the combination with parallel injection of the higher energy beam. The results obtained from Eq.(16) are shown by circles for comparison, which have almost the same profile as those obtained by the 2D code for the case of nearly-isotropic injection.

5. CONCLUSION

We have investigated the fusion reactivity enhancement due to the ion tail formation by NBI and second harmonic ICH in the 50:50 D-T plasma on the basis of the local Fokker-Planck calculation. From the viewpoint of the fusion reactivity enhancement, we have compared the beam-induced tail with the RF-induced one, and examined the combined effects of them.
magnitude, as compared with $<\sigma v>_{\eta}$. The rate of increase of the reactivity with the deposited power density, $\dot{P}_d$, for only NBI is deteriorated by RF injection in the combination of parallel injection of high energy beam and RF heating (dashed lines). For $E_0 = 200 \text{ keV}$, even the combination of the perpendicularly injected beam and RF is not advantageous, and the increase in the beam power density is preferable. This is because RF-enhancement of the beam-induced tail in the higher velocity region beyond the peak point of $C(v)$ does not work very effectively in the reactivity enhancement. On the contrary, the reactivity enhanced by the perpendicularly injected $100\text{ keV}$ beam can be further improved by RF injection. In this case RF-induced velocity diffusion works effectively by producing the higher energy tail which is given access to the '$\sigma v$-function' $G(v)$ in the region closer to its peak point.

The improvement in the reactivity by the RF wave is realized when the ion tail is formed by the perpendicularly injected $100\text{ keV}$ beam. The reactivity enhancement factor, $\eta$, is shown in Fig.11 as a function of $P_{\text{NBI}}/P_d$ for the combination of parallel (cross) or perpendicular (square) injection of $100\text{ keV}$ beam and RF heating with $\dot{P}_d = 10$ for $T = 5\text{keV}$. Note that the optimum fraction of power density supplied by the perpendicular beam exists; $P_{\text{NBI}}/P_d \sim 0.7$ for this case. The value of $\eta$ at $P_{\text{NBI}}/P_d = 1$ and the optimum fraction become small with the decrease of $E_0$. For $E_0 = 50 \text{ keV}$ $\eta$ decreases with the increase of $P_{\text{NBI}}/P_d$. On the other hand, the reactivity is degraded by RF injection in the combination with parallel injection of the higher energy beam. The results obtained from Eq.(16) are shown by circles for comparison, which have almost the same profile as those obtained by the 2D code for the case of nearly-isotropic injection.

5. CONCLUSION

We have investigated the fusion reactivity enhancement due to the ion tail formation by NBI and second harmonic ICH in the 50:50 D-T plasma on the basis of the local Fokker-Planck calculation. From the viewpoint of the fusion reactivity enhancement, we have compared the beam-induced tail with the RF-induced one, and examined the combined effects of them.
Analytic results as well as numerical results show the following:

The difference between the beam-induced tail and the RF-induced one is clearly seen in $\nu^2 F_0(\nu) G(\nu)$ ($F_0(\nu)$ is the angle-averaged distribution function and $G(\nu)$ the 'σν-function'). The profile of $\nu^2 F_0(\nu) G(\nu)$ typically presents two humps for NBI heating. One is due to the bulk component, and the other is due to the beam-induced tail. Thus the beam injection with $E_0$ larger than the energy at the peak of $G$, $E_\nu \sim 130$ keV, is effective for the reactivity enhancement. The optimum $E_0$ value for various $T_e$ and $T_i$ can be easily found by using Eqs. (32) and (34). On the other hand, the RF-induced tail brings a 'wing' rather than a hump, and the reactivity enhancement is smaller than that for NBI with the adequate injection energy. When RF heating is combined with NBI heating, the ion tail spreads to the higher energy region, $E > E_0$. Therefore, the reactivity is enhanced by $P_{RF}$ more effectively than by $P_{NBI}$ with $E_0 < E_\nu$, while the enhancement by $P_{RF}$ is smaller than that by $P_{NBI}$ with $E_0 > E_\nu$.

For the case that $E_0$ is a little lower than $E_\nu$ ($E_0 = 100$ keV), the combination of the perpendicularly injected beam and the RF wave is effective for the reactivity enhancement. The rate of increase of the reactivity with the deposited power density can be improved by RF heating as compared with that by only NBI heating. When the beam is injected parallel to the magnetic field, the efficiency of the reactivity enhancement is deteriorated by RF heating as compared with the case of only NBI heating. Thus, in the present work, the basic aspects of TCT-enhancement of the reactivity by ion tail formation due to NBI and second harmonic ICH have been indicated together with the possibility that the combined effects of them can exceed the each effect on the reactivity enhancement in the efficiency.

We discuss about the fusion power multiplication factor, Q. If we define Q as $Q = P_f/P_d$, where $P_f$ is the fusion power density proportional to $<\sigma v>$. Q value is decreased by the increase of $P_d$ in the present model. This is because the bulk plasma temperature is taken to be constant and $P_f$ has a value proportional to $<\sigma v>$ without any heating power. The increase in the electron and triton temperature is preferable in the reactivity enhancement and hence in the increase of Q; the former leads to the
increase in the slowing down time of the high energy deuteron and the latter results in the enhancement of the '\sigma\nu-function' \( C(\nu) \). However, the distributions of electrons and tritons are assumed to be Maxwellian with the constant temperature in spite of the increase in the rates of collisional energy transfer from deuterons to electrons and tritons, which comes from the deformation of the deuteron distribution. This means that the energy confinement times of electrons and tritons are substantially decreased with the increase in the deposited NB and RF power density. The confinement time of the plasma is almost inversely proportional to \( P_d \). In order to evaluate \( Q \) value, we have to take into account of the energy confinement time for each species. Here we introduce another kind of multiplication factor defined as \( Q_\nu = (P_f - \langle P_f \rangle_\nu)/P_d \), where \( \langle P_f \rangle_\nu \propto \langle \sigma \nu \rangle_\nu \) is the fusion power density for the Maxwellian plasma and 17.6MeV energy is assumed to be released by one fusion reaction. The value of \( Q_\nu \) at the optimum \( E_0 \) value obtained from Eqs. (32) and (34) is about \( Q_\nu = 0.2 \) for \( T_e = T_i = 2\)keV, \( Q_\nu = 0.55 \) for \( T = 5\)keV, \( Q_\nu = 0.8 \) for \( T = 10\)keV, and \( Q_\nu = 0.7 \) for \( T = 20\)keV.

The triton tail formation by the third harmonic triton cyclotron heating, which is brought by a wave with the frequency of \( \omega = 2\omega_{ce} = 3\omega_{ct} \), may lead to the reactivity enhancement in large \( k_{\perp D_{ih}} \). On the contrary, the direct loss of the high energy ions deteriorates the reactivity. Spatial effects such as inhomogeneity of the beam deposition and the wave absorption should be considered for strict evaluation of \( Q \). Effects of magnetically trapped ions on the quasi-linear RF diffusion term are also important. These problems are left for future study.

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increase in the slowing down time of the high energy deuteron and the latter results in the enhancement of the \( g_t \)-function \( G(v) \). However, the distributions of electrons and tritons are assumed to be Maxwellian with the constant temperature in spite of the increase in the rates of collisional energy transfer from deuterons to electrons and tritons, which comes from the deformation of the deuteron distribution. This means that the energy confinement times of electrons and tritons are substantially decreased with the increase in the deposited NB and RF power density. The confinement time of the plasma is almost inversely proportional to \( P_d \). In order to evaluate \( Q \) value, we have to take into account of the energy confinement time for each species. Here we introduce another kind of multiplication factor defined as \( Q_s = \langle P_f - \langle P_f \rangle_H \rangle / P_d \), where \( \langle P_f \rangle_H \propto \langle \sigma U \rangle_H \) is the fusion power density for the Maxwellian plasma and 17.6MeV energy is assumed to be released by one fusion reaction. The value of \( Q_s \) at the optimum \( E_0 \) value obtained from Eqs. (32) and (34) is about \( Q_s = 0.2 \) for \( T = T_e = T_i = 2\)keV, \( Q_s = 0.55 \) for \( T = 5\)keV, \( Q_s = 0.8 \) for \( T = 10\)keV, and \( Q_s = 0.7 \) for \( T = 20\)keV.

The triton tail formation by the third harmonic triton cyclotron heating, which is brought by a wave with the frequency of \( \omega = 2\omega_B = 3\omega_T \), may lead to the reactivity enhancement in large \( k_B D_{th} \). On the contrary, the direct loss of the high energy ions deteriorates the reactivity. Spatial effects such as inhomogeneity of the beam deposition and the wave absorption should be considered for strict evaluation of \( Q \). Effects of magnetically trapped ions on the quasi-linear RF diffusion term are also important. These problems are left for future study.

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FIGURE CAPTIONS

Fig. 1 Deposited RF power density, $\dot{P}_{RF}$, versus $\dot{P}_{B1}$, for cases of only ICH (triangle) and NBI + ICH (circle) with $\dot{P}_{B1} = 2$, $E_0/T = 20$, and $k_wE_0/c\omega_D = 0.9$. Dashed line denotes $\dot{P}_{RF} > 0$.

Fig. 2 Profiles of (A) $G(v)$ in unit of cm$^2$ and (B) $v^2F_0(v)G(v)$ for Maxwellian D-T plasma with temperature of (i) 5keV and (ii) 10keV. $v$ is normalized by $v_{ih}$.

Fig. 3 Dependence of reactivity, $<\alpha u>$, on temperature, $T$ for Maxwellian plasma (dashed line) and for plasma with beam-induced tail (solid line). Deposited NB power density is constant as $\dot{P}_{B1} = 10$ at $T = 5$keV, and injection energy is (a) $E_0 = 50$keV, (b) 100keV, and (c) 200keV. Black circles denote results of Eqs. (32) and (34).

Fig. 4 Profiles of (A) $F_0(v)$ and (B) $v^2F_0(v)G(v)$ for only NBI with (a) $E_0 = 50$keV, (b) 100keV, (c) 200keV, and (d) for only RF heating. Heating power density is $\dot{P}_{B1} = 2$ (dashed line) and $\dot{P}_{RF} = 5$ (solid line) for cases (a-c), and $\dot{P}_{RF} = 2.1$ (dashed line) and $\dot{P}_{RF} = 5$ (solid line) for case (d). Plasma temperature is 5keV. Two humps are seen in $v^2F_0(v)G(v)$ for NBI heating, while 'wing' is formed for RF heating.

Fig. 5(i) Reactivity enhancement factor, $\eta = <\alpha u>/<\alpha u>_0$, versus deposited power density, $\dot{P}_d$, for $T = 5$keV. Injection energy is (a) $E_0 = 50$keV (cross), (b) 100keV (circle), and (c) 200keV (square). Triangles correspond to case of (d) only RF heating. When RF heating is added to NBI heating with $\dot{P}_{B1} = 6$ (dashed line), rate of $\eta$ increase becomes small for $E_0 = 200$keV, and it is almost the same for $E_0 = 100$keV as that by only NBI heating.
Fig. 5(ii) Reactivity enhancement factor, \( \eta = \langle \alpha v \rangle / \langle \alpha v \rangle_{\nu} \), versus deposited power density, \( \tilde{\rho}_d \) for \( T = 10 \) keV. Injection energy is (a) \( E_0 = 50 \) keV (cross), (b) \( 100 \) keV (circle), and (c) \( 200 \) keV (square). Triangles correspond to case of (d) only RF heating. When RF heating is added to NBI heating with \( \tilde{\rho}_{\nu} = 8.3 \) (dashed line), rate of \( \eta \) increase becomes small for \( E_0 = 200 \) keV, and it is almost the same for \( E_0 = 100 \) keV as that by only NBI heating.

Fig. 6 Profiles of (A) \( F_0(v) \) and (B) \( v^2 F_0(v) G(v) \) for combination of NBI and RF heating \( (\tilde{\rho}_{\nu} = 2 \) and \( \tilde{K}_2 = 1 \) \). Beam energy is (a) \( E_0 = 50 \) keV, (b) \( E_0 = 100 \) keV, and (c) \( E_0 = 200 \) keV. Plasma temperature is 5 keV.

Fig. 7 Numerical results of (A) \( F_0(v) \) and (B) \( v^2 F_0(v) G(v) \). Case (a) is only NBI heating and cases (b-d) are combination with RF heating \( (\tilde{K}_2 = 1) \). For all cases, beam power density and beam energy are \( \tilde{\rho}_{\nu} = 2 \) and \( E_0 = 100 \) keV, respectively. While injection angle is varied; (b) parallel injection to magnetic field, (c) perpendicular injection, and (d) nearly-isotropic injection. Plasma temperature is 5 keV.

Fig. 8 Profiles of (A) \( F_0(v) \) and (B) \( v^2 F_0(v) G(v) \) for only RF heating of \( \tilde{\rho}_{RF} = 5 \). Coefficients of Legendre expansion, \( F_0(v) \) (solid line), \( F_2(v) \) (dashed line), and \( F_4(v) \) (dash-dotted line) are shown in (C).

Fig. 9 Deposited RF power density, \( \tilde{\rho}_{RF} \), versus \( \tilde{K}_2 \) for only RF heating (triangle) and in combination of RF heating and 100 keV beam injection with \( \tilde{\rho}_{\nu} = 2 \). Cases of parallel (cross), perpendicular (square), and nearly-isotropic (circle) injections are indicated. Perpendicularly injected beam couples the most with RF waves. \( \langle \tilde{\rho}_{RF} \rangle_{\nu} \) is also shown by dashed line.
Fig. 10 Reactivity enhancement factor, $\eta = \langle \sigma t \rangle / \langle \sigma t \rangle_{0}$, versus deposited power density, $\dot{P}_d = \dot{P}_{NB} + \dot{P}_{RF}$, for $T = 5$ keV. Only NBI cases are shown by solid lines with (a) crosses ($E_0 = 50$ keV), (b) circles ($E_0 = 100$ keV), and (c) squares ($E_0 = 200$ keV). Only RF case (d) is shown by dash-dotted line with triangles. Dashed lines correspond to cases of combination of parallel beam injection and RF heating, and dotted lines to cases of combination of perpendicular beam injection and RF heating.

Fig. 11 Reactivity enhancement factor, $\eta$, versus $P_{NB}/P_d$ for combination of parallel (cross) or perpendicular (square) injection of 100 keV beam and RF heating with $\dot{P}_d = 10$ for $T = 5$ keV. Results obtained from Eq. (16) are shown by circles for comparison.
Fig. 1 Deposited RF power density, $\dot{P}_{\text{RF}}$, versus $\dot{k}_2$, for cases of only ICH (triangle) and NBI + ICH (circle) with $\dot{A}_{\text{BH}} = 2$, $E_0/T = 20$, and $k_{\perp v_0}/c_0 = 0.9$. Dashed line denotes $\langle \dot{P}_{\text{RF}} \rangle_N$. 

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Fig. 5(ii) Reactivity enhancement factor, $\eta = \langle \omega t \rangle / \langle \omega t \rangle_N$, versus deposited power density, $\dot{P}_d$ for $T = 10\text{keV}$. Injection energy is (a) $E_0 = 50\text{keV}$ (cross), (b) $100\text{keV}$ (circle), and (c) $200\text{keV}$ (square). Triangles correspond to case of (d) only RF heating. When RF heating is added to NBI heating with $\dot{P}_{\text{RF}} = 8.3$ (dashed line), rate of $\eta$ increase becomes small for $E_0 = 200\text{keV}$, and it is almost the same for $E_0 = 100\text{keV}$ as that by only NBI heating.
Fig. 6 Profiles of (A) $F_0(v)$ and (B) $v^2F_0(v)G(v)$ for combination of NBI and RF heating ($R_{\text{NBI}} = 2$ and $R_C = 1$). Beam energy is (a) $E_0 = 50\text{keV}$, (b) $E_0 = 100\text{keV}$, and (c) $E_0 = 200\text{keV}$. Plasma temperature is 5keV.
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Fig. 10 Reactivity enhancement factor, $\eta = \langle N \tau \rangle / \langle N \tau \rangle_d$, versus deposited power density, $\tilde{P}_d = \tilde{P}_\text{NI} + \tilde{P}_\text{RF}$, for $T = 5\text{keV}$. Only NBI cases are shown by solid lines with (a) crosses ($E_0 = 50\text{keV}$), (b) circles ($E_0 = 100\text{keV}$), and (c) squares ($E_0 = 200\text{keV}$). Only RF case (d) is shown by dash-dotted line with triangles. Dashed lines correspond to cases of combination of parallel beam injection and RF heating, and dotted lines to cases of combination of perpendicular beam injection and RF heating.
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