Relaxation function method
for evaluation of general thermal loads

(Research Report)

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2000
Relaxation function method for evaluation of general thermal loads

(Research Report)

Naoto Kasahara*

Abstract

A rational analysis method for general thermal transient problems was proposed, by utilizing transient response characteristics of structures. Structures can not respond to quick change of fluid temperature, since they have finite time constants. Low rate changes of fluid temperature hardly induce thermal stresses because of temperature homogenization in structures. In order to quantify above transient characteristics, author developed a relaxation function method that describes Green function of structures. This function can be applied to sensitivity analysis of thermal stress to fluid temperature and to optimization of heat removal systems.

* Structure and Material Research Group, System Engineering Division, OBC, JNC
一般的熱通過問題を解析するための緩和関数法

（研究報告書）

笠原 直人*

要 旨

構造の過渡応答特性を利用することによって、一般的な熱通過問題に対する合理的な構造解析法を提案した。構造物は固有の応答時定数を有するため、急激な流体温度変化に追従することは不可能である。逆に緩やかな温度変化は構造内部の熱伝導によって均熱化されるため、熱応力の要因となる構造内温度勾配を生じさせ難い。

過渡的温度変化に対する上記特性を定量化するため、本研究では構造のステップ応答を記述する緩和関数法を開発した。

本関数は、熱応力の流体温度変化に対する感度解析と、プラント冷却系の最適構造設計に適用することが出来る。

尚、本内容は1999年9月から2000年8月までの期間にCEAカダラッシュ研究所にて実施した業務の一部である。

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</tbody>
</table>
NOMENCLATURE

$T_f(t)$: Temperature of fluid
$T_s(x,t)$: Temperature of structure
$\sigma(x,t)$: Stress in structure
$G(x,t)$: Time response function of structure to fluid temperature fluctuation
$H(t)$: Time response function of effective heat transfer
$S(x,t)$: Time response function of effective thermal stress
$\phi[T_s(x,t)]$: Thermal stress function determined by mechanical boundary conditions
$T_f(s)$: Laplace transform of $T_f(t)$
$T_s(x,s)$: Laplace transform of $T_s(x,t)$
$G(x,s)$: Laplace transform of $G(x,t)$
$H(s)$: Laplace transform of $H(t)$
$S(x,s)$: Laplace transform of $S(x,t)$
$\Phi[T_s(x,s)]$: Laplace transform of $\phi[T_s(x,t)]$
$\tau$: Time constant of structural temperature response
$\zeta$: Igen values of structures
$x$: Length from the surface of structure
$t$: Time
$h$: Heat transfer coefficient
$L$: Wall thickness of structure
$A$: Area
$V$: Volume
$a$: Thermal diffusivity of structural material
$\lambda$: Heat conductivity of structural material
$c$: Specific heat
$\rho$: Density
$E$: Young's modulus of structural material
$\alpha$: Linear expansion coefficient of structural material
$\nu$: Poisson's ratio of structural material
$K$: Stress index determined by mechanical boundary conditions and material properties
$K = 1/(1-\nu)$ in the case of biaxial plane stress condition
1 INTRODUCTION

Design experiences of Japanese DFBR notified that thermal transient loads induced by fluid temperature change become sometimes critical for their plant design. Thermal transient loads are caused from combined thermohydraulic and thermomechanical phenomena as in Fig.1.1, so that there are neither rational evaluation methods nor rules for these problems. Recently, simple and compact components are demanded for FBRs because of a strong requirement of economical improvement, which leads to increase thermal stresses. On the otherhand, seismic loads give no limitation because of adoption of seismic isolation systems. This situation motivated author to develop total assessment approaches of thermohydraulic and thermomechanical phenomena. One of them is a total simulation of fluids and structures by the Object oriented code (Figs.1.2, 1.3 [1]).

Another is a theoretical approach with structural response functions by utilizing transient response characteristics of structures as in Fig.1.4. Structures can not respond to quick change of fluid temperature, since they have finite time constants. Low rate changes of fluid temperature hardly induce thermal stresses because of temperature homogenization in structures. These characteristics vary according to structures as in Fig.1.5. In order to quantify above transient characteristics, author developed a relaxation function method with time constants of structures. By using relaxation functions, we can understand characteristics of Green function clearly. For design of the SPX plant, French engineers developed an assessment method of time dependent damage, where they defined the forgetting time that is similar to the time constant of relaxation function [2] [3]. For example, when a time constant of fluid temperature change is longer than structural one, its structure forgets effect from fluid. Another application of this method is quantification of Green function that can be applied to fast calculation of transient thermal stress as in Fig.1.6 [1]. Green function is also utilized for on-line monitoring [4]. The purpose of this study is a development of the relaxation function method for general thermal transient problems by extension of the frequency response function [5] [6].
Fig.1.1 Thermal transient phenomena due to plant dynamics

Fig.1.2 Evaluation procedures of thermal transient strength of nuclear components
Fig. 1.3 Graphical user interface of the total simulation code PARTS

Changing rate of fluid temperature  Attenuation mechanism  Induced thermal stress amplitude

Heat transfer losses

High \[\rightarrow\] from fluid to structures \[\rightarrow\] Low

Medium \[\rightarrow\] Maximum

Low \[\rightarrow\] Thermal homogenization inside structures \[\rightarrow\] Low

Fig. 1.4 General tendency of structural response to fluid temperature fluctuation
Fig. 1.5 Variety of structural response characteristics to the same thermal transient condition.

Fig. 1.6 Fast stress calculation with Green function.
2. FORMULATION OF PHENOMENA BY RELAXATION FUNCTIONS

2.1. DESCRIPTION OF PHENOMENA AS RESPONSE PROBLEMS

Structural surfaces that contact to fluid can respond rapidly to fluid temperature change as in Fig. 2.1. Temperature within structures follow slowly to surface temperature. Different response characteristics among structural part cause temperature distribution and thermal stress. Temperature response characteristics depend on structural configuration and they can be quantified by relaxation functions.

Fig. 2.1 Mechanism of transient thermal stress induced by fluid temperature change
2.2. ONE DEGREE OF HEAT CAPACITY PROBLEM

In order to understand mechanism and to quantify structural response characteristics, thermal and mechanical responses problems are theoretically formulated. Fig.2.2 is a one degree of heat capacity model. A differential equation with a boundary condition for this problem is

\[ Vc_\rho T_s' + Ah T_s = Ah T_f \]  \hspace{1cm} (2.1)

Laplace transform of Eq.(2.1) is

\[ (Vc_\rho s + Ah)T_s(s) = Ah T_f(s). \] \hspace{1cm} (2.2)

Transfer function \( H(s) \) from fluid temperature \( T_f(s) \) to structural temperature \( T_s(s) \) can be described as

\[ H(s) = \frac{T_s(s)}{T_f(s)} = \frac{1}{1+\frac{s}{\tau}}, \quad \tau = \frac{Vc_\rho}{Ah}. \] \hspace{1cm} (2.3)

Response of this system to a step temperature change is

\[ U_H(s) = H(s)X(s) = \frac{1}{1+\frac{s}{\tau}} \cdot \frac{1}{s} \] \hspace{1cm} (2.4)

that is the first order delay system with the time constant \( \tau \).

Inverse Laplace transform of Eq.(2.4) gives

\[ T_s(t) = 1 - e^{-t/\tau} \] \hspace{1cm} (2.5)

which is the relaxation function with the relaxation time \( \tau \).
Figure 2.2 One degree of heat capacity problem
2.3. TWO DEGREES OF HEAT CAPACITY PROBLEM

Next problem described in Fig.2.3 is a two degrees of heat capacity model. In this problem, thermal stress is assumed to be proportional to difference between two heat capacity points as

\[ \sigma = KE\alpha (T_{x0} - T_{x1}) \],

(2.6)

where K is a stress factor determined from a geometry.

Differential equation with boundary condition is

\[
\begin{bmatrix}
K_{x0}\rho_{x0} & 0 \\
0 & K_{x1}\rho_{x1}
\end{bmatrix}
\begin{bmatrix}
T_{x0} \\
T_{x1}
\end{bmatrix} +
\begin{bmatrix}
\frac{A_{x1}h}{L_{1}} + \frac{A_{x1}}{L_{1}} & -\frac{A_{x1}}{L_{1}} \\
-\frac{A_{x1}}{L_{1}} & \frac{A_{x1}}{L_{1}}
\end{bmatrix}
\begin{bmatrix}
T_{x0} \\
T_{x1}
\end{bmatrix} =
\begin{bmatrix}
A_{x0}h & 0 \\
0 & A_{x0}
\end{bmatrix}
\begin{bmatrix}
T_{x0} \\
T_{x1}
\end{bmatrix},
\]

(2.7)

where \( A_{0} \) is a convection area, \( A_{1} \) is conductive area, \( L_{1} \) is conductive length with heat conductivity \( \lambda_{1} \).

Transfer function from fluid temperature to stress can be obtained by Laplace transform of Eqs.(2.6) and (2.7) as

\[ \frac{\sigma(s)}{T_{f}(s)} = \frac{T_{x0}(s)}{T_{f}(s)} \cdot \frac{\sigma(s)}{T_{x0}(s)} = KE\alpha H(s)S(s) = KE\alpha G(s). \]

(2.8)

In Eq.(2.8), \( H(s) \) is an effective heat transfer function described as

\[ H(s) = \frac{T_{x0}(s)}{T_{f}(s)} = \frac{1}{1 + \tau_{f} s}, \quad \tau_{f} = \frac{V_{0}c_{0}\rho_{0}}{A_{0}h}. \]

(2.9)

which is the same as Eq.(2.5), and

S(s) is an effective thermal stress function expressed as

\[ S(s) = \frac{1}{KE\alpha T_{x0}(s)} \cdot \frac{\sigma(s)}{1 + \tau_{s} s}, \quad \tau_{s} = \frac{V_{1}c_{1}\rho_{1}L_{1}}{A_{1}\lambda_{1}}. \]

(2.10)

When inputting the relaxation function
\[ T(t) = e^{-t \tau}, \quad (2.11) \]

to Eq. (2.7), a determination equation

\[ \det \left[ Vcp + \left[ \frac{A \lambda}{L} \right] \right] = 0 \quad (2.12) \]

is obtained and this solutions are two eigen values \( \tau_1, \tau_2 \) and two eigen vectors \( \phi_1, \phi_2 \).

After that, time response of the two degree system is

\[ T = a_1 e^{\tau_1 t} + a_2 e^{\tau_2 t}. \quad (2.13) \]
\[
\begin{align*}
\sigma(s) &= \frac{T_{so}(s)}{T_f(s)} \cdot \frac{\sigma(s)}{T_{so}(s)} \cdot \left\{ \begin{array}{c}
H(s) = \frac{T_{so}(s)}{T_f(s)} = \frac{1}{1 + \tau_f s} \\
S(s) = \frac{1}{KE\alpha T_{so}(s)} = \frac{1}{1 + \tau_s s} \\
G(s) \rightarrow 0
\end{array} \right.
\end{align*}
\]

\[
\begin{align*}
\sigma(s) &= \frac{K\varepsilon_0 \rho_0}{A_0} \frac{V_{0} \varepsilon_0 \rho_0}{L_0} \frac{V_{1} \varepsilon_1 \rho_1}{L_1} \frac{A_{0} h}{L_0} \frac{A_{1} h}{L_1} \frac{A_{2} h}{L_2} \frac{A_{3} h}{L_3} \frac{A_{4} h}{L_4} \\
\sigma &= KE \alpha (T_{so} - T_{sl})
\end{align*}
\]

\[
\begin{align*}
U_{sl}(s) &= \frac{V_{0} \varepsilon_0 \rho_0}{A_0} \frac{V_{1} \varepsilon_1 \rho_1}{A_1} \frac{V_{2} \varepsilon_2 \rho_2}{A_2} \frac{V_{3} \varepsilon_3 \rho_3}{A_3} \frac{V_{4} \varepsilon_4 \rho_4}{A_4} \frac{V_{5} \varepsilon_5 \rho_5}{A_5} \frac{V_{6} \varepsilon_6 \rho_6}{A_6} \\
U_{e}(s) &= \frac{\sigma(s)}{2} \frac{K\varepsilon_0 \rho_0}{A_0} \frac{K\varepsilon_1 \rho_1}{A_1} \frac{K\varepsilon_2 \rho_2}{A_2} \frac{K\varepsilon_3 \rho_3}{A_3} \frac{K\varepsilon_4 \rho_4}{A_4} \frac{K\varepsilon_5 \rho_5}{A_5} \frac{K\varepsilon_6 \rho_6}{A_6}
\end{align*}
\]

\[
\begin{align*}
T_{so} &= T_f \left( 1 - e^{-\tau_f t} \right) \\
T_{sl} &= T_f \left( 1 - \frac{\tau_f^2}{\tau_f - \tau_s} e^{-\tau_s t} + \frac{\tau_f^2}{\tau_f - \tau_s} e^{-\tau_f t} \right) \\
\sigma &= KE \alpha \left( e^{-\tau_s t} + \frac{\tau_f^2}{\tau_f - \tau_s} e^{-\tau_s t} - \frac{\tau_f^2}{\tau_f - \tau_s} e^{-\tau_f t} \right)
\end{align*}
\]
3. METHOD TO GET RELAXATION FUNCTIONS

3.1. MODAL ANALYSIS METHOD

The formulation of two degree system can be extended to N-dimensional problems as the next equations.

\[
\begin{bmatrix} V_r \rho \end{bmatrix} + \left[ \frac{AL}{L} \right] \begin{bmatrix} T \end{bmatrix} = \begin{bmatrix} q \end{bmatrix}
\]  

(3. 1)

\[
\text{det} \left[ \begin{bmatrix} V_c \rho \end{bmatrix} + \left[ \frac{AL}{L} \right] \right] = 0
\]

(3. 2)

Above equation gives n eigen values \( \tau_1, \tau_2, \cdots, \tau_n \), and n eigen vectors \( \phi_1, \phi_2, \cdots, \phi_n \).

After that, time response of the n degree system is

\[
T = a_1 e^{\tau_1 t} + a_2 e^{\tau_2 t} + \cdots + a_n e^{\tau_n t}
\]

(3. 3)

\( a_n \) factors are determined From Eq.(3.2)

\[
(\tau_i - \tau_j) \left( \phi_i \right)^T \left[ C \right] \left( \phi_j \right) = 0,
\]

(3. 4)

that has orthogonal properties.

Modal analysis programs for elastic structures may be utilized to above calculation with some modification.

A detailed formulation for programming is required as a future study.
3.2. THEORETICAL ANALYSIS METHOD

Theoretical analysis approach is applicable for simple problems.
For example, consider a plate of infinite extent, which is subjected to a step
temperature change of fluid on its front surface through heat transfer coefficient $h$
with the back surface being adiabatic. The through thickness temperature can be
found from the one dimensional heat conduction equation.

$$\frac{\partial T_s}{\partial t} = a \frac{\partial^2 T_s}{\partial x^2}, \quad a = \frac{\lambda}{c\rho}$$  \hspace{1cm} (3.5)

Under the boundary condition:

$$T_s(0,t) = 0$$  \hspace{1cm} (3.6)

$$T_s(1,t) = 1$$  \hspace{1cm} (3.7)

and the initial condition:

$$T_s(x,0) = 0,$$  \hspace{1cm} (3.8)

Solution[7] is

$$T_s(x,t) = \sum_{n=1}^{\infty} a_n e^{-\lambda_n x} \sin(\lambda_n x),$$  \hspace{1cm} (3.9)

where

$$\tan \lambda_n = -\frac{1}{h},$$  \hspace{1cm} (3.10)

$$a_n = \frac{2\lambda_n}{\lambda_n - \sin \lambda_n \cos \lambda_n} \int_0^1 \sin(\lambda_n \xi) d\xi$$  \hspace{1cm} and  

$$\int_0^1 \sin(\lambda_n \xi) \sin(\lambda_m \xi) d\xi = \begin{cases} 1/2, & (m = n) \\ 0, & (m \neq n) \end{cases}$$  \hspace{1cm} (3.11)

Once temperature distribution $T_s(x,t)$ was obtained, the following equations[8]
give stress distribution $\sigma(x,t)$.  

---

- 13 -
\( \sigma(x,t) = -K_m M(t) - K_s [3M(t) - B(t)] \left( 1 - \frac{2x}{L} \right) - K_p \left[ T_s(x,t) - \left( 4 - \frac{6x}{L} \right) M(t) + \left( 1 - \frac{2x}{L} \right) B(t) \right] \)  

(3.13)

\[ M(t) = \frac{1}{L} \int_0^L T_s(x,t) \, dx \]  

(3.14)

\[ B(t) = \frac{1}{L^2} \int_0^L x T_s(x,t) \, dx \]  

(3.15)

\[ K_m = K_s = K_p = K = \frac{E \alpha}{1 - \nu} \]

\[ K_m = K_s = K = \frac{E \alpha}{1 - \nu} \]

\[ K_m = K_s = K = \frac{E \alpha}{1 - \nu} \]

\[ K_m = K_s = K_p = K = \frac{E \alpha}{1 - \nu} \]

\[ K_m = K_s = K_p = K = \frac{E \alpha}{1 - \nu} \]

Fig. 3.2 One dimensional continuum problem
3.3 REGRESSION ANALYSIS METHOD

Green functions of structures can be described by linear summation of relaxation functions as

\[ U(x,t) = \sum_{n=1}^{\infty} T_n(t) X_n(x) \]  \hspace{1cm} (3.16)

time term \( T_n(t) = e^{-\zeta_n^2 t} \) \hspace{1cm} (3.17)

spatial term \( X_n(x) = \alpha_n \sin(\zeta_n x) \) \hspace{1cm} (3.18)

Paying attention to above expression, it is possible to approximate Green functions by linear summation of several relaxation functions, time constant of which covers the phenomena. Factors of relaxation functions are determined by regression analysis.

Since time constants of thermal transient phenomena in nuclear plants exist between 0.2 seconds and 10000 seconds, the next equation can approximate phenomena.

\[ \sigma(t) = \sum_{n=1}^{15} C_n e^{-t/T_n} \]  \hspace{1cm} (3.19)

where \( C_n \) is a factor for relaxation functions.

**Table 3.1 Time constants for 15 order approximation by eigenfunction**

<table>
<thead>
<tr>
<th>Order</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_n ) sec</td>
<td>10000</td>
<td>5000</td>
<td>2000</td>
<td>1000</td>
<td>500</td>
<td>200</td>
<td>100</td>
<td>50</td>
<td>20</td>
<td>10</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>0.5</td>
<td>0.2</td>
</tr>
</tbody>
</table>
To validate this idea, pipes subjected to step temperature change of fluid are considered. Material is 316FR SS and ratio of half diameter to wall thickness is kept R/t=10. Heat transfer coefficient is 5000 kcal/m²h°C. Wall thickness are 2mm, 5mm, 10mm, 20mm, 50mm, 70mm and 100mm. The next figure shows F.E. calculated stress histories. These results explain that thicker wall pipes generate larger stress and keep them longer time.

![Graph](image)

**Fig.3.3** Green's functions of pipes with various wall thickness (h=5000kcal/m²°C)

When approximating stress histories of Fig.3.3 by Eq.(3.19), results have good agreement with original curves. Obtained factors by regression analysis are shown in Table 3.2 and in Fig.3.4. Factors of relaxation functions with long time constants increase when wall thickness increase. From these results, it was clarified that factors of relaxation functions quantified Green functions.
Table 3.2 Coefficients of eigenfunctions for Pipes with various wall thickness

<table>
<thead>
<tr>
<th>$\tau_n, \text{sec}$</th>
<th>$t=2\text{mm}$, $h=5000\text{kcal}$</th>
<th>$t=5\text{mm}$, $h=5000\text{kcal}$</th>
<th>$t=7\text{mm}$, $h=5000\text{kcal}$</th>
<th>$t=10\text{mm}$, $h=5000\text{kcal}$</th>
<th>$t=20\text{mm}$, $h=5000\text{kcal}$</th>
<th>$t=50\text{mm}$, $h=5000\text{kcal}$</th>
<th>$t=70\text{mm}$, $h=5000\text{kcal}$</th>
<th>$t=100\text{mm}$, $h=5000\text{kcal}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>-0.0002</td>
<td>0.0002</td>
<td>0.0003</td>
<td>0.0004</td>
<td>0.0005</td>
<td>0.0006</td>
<td>0.0003</td>
<td>0.0004</td>
</tr>
<tr>
<td>5000</td>
<td>0.0005</td>
<td>0.0007</td>
<td>0.0006</td>
<td>0.0001</td>
<td>0.0008</td>
<td>0.0017</td>
<td>0.0034</td>
<td>0.0045</td>
</tr>
<tr>
<td>2000</td>
<td>0.0010</td>
<td>0.0014</td>
<td>-0.0011</td>
<td>0.0025</td>
<td>0.0017</td>
<td>0.0273</td>
<td>0.0106</td>
<td>0.0262</td>
</tr>
<tr>
<td>1000</td>
<td>0.0017</td>
<td>0.0022</td>
<td>0.0018</td>
<td>-0.0039</td>
<td>0.0028</td>
<td>-0.0527</td>
<td>-0.0295</td>
<td>0.3594</td>
</tr>
<tr>
<td>500</td>
<td>-0.0019</td>
<td>0.0024</td>
<td>-0.0020</td>
<td>0.0044</td>
<td>-0.0033</td>
<td>0.0973</td>
<td>0.3842</td>
<td>0.0058</td>
</tr>
<tr>
<td>200</td>
<td>0.0026</td>
<td>0.0034</td>
<td>0.0029</td>
<td>-0.0067</td>
<td>0.0060</td>
<td>0.3639</td>
<td>0.0212</td>
<td>-0.0244</td>
</tr>
<tr>
<td>100</td>
<td>-0.0042</td>
<td>-0.0056</td>
<td>-0.0048</td>
<td>0.0122</td>
<td>-0.0150</td>
<td>-0.1430</td>
<td>-0.0149</td>
<td>0.0938</td>
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<td>-0.0780</td>
<td>-0.1008</td>
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<td>-0.1235</td>
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<td>-0.1433</td>
<td>-0.1265</td>
<td>-0.1100</td>
<td>-0.0864</td>
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</table>

Fig.3.4 Structural response characteristics to time constant (h=5000kcal/m²h°C)

To be compared, time when stress becomes the maximum and time constant when stress becomes the maximum stress divided by e were evaluated as in the next table and figure. This tendency agrees with Green function results. The later can provide more detailed information such as the maximum stress level.
Table 3.3 Time constants for Pipes (h=5000 kcal/mm²·h°C)

<table>
<thead>
<tr>
<th>t (mm)</th>
<th>2</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>70</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ max sec</td>
<td>0.4</td>
<td>0.7</td>
<td>1.5</td>
<td>3</td>
<td>6</td>
<td>16</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>τ 1/2 sec</td>
<td>6</td>
<td>7</td>
<td>12</td>
<td>20</td>
<td>60</td>
<td>280</td>
<td>550</td>
<td>1000</td>
</tr>
</tbody>
</table>

Fig. 3.5 Time constants of pipes (h=5000 kcal/m²·h°C)

The second problem is pipes subjected to step temperature change of fluid with different heat transfer coefficients. Material is 316FR SS, half diameter is 100mm and wall thickness is 10mm. Heat transfer coefficients are 1000 kcal/m²·h°C, 2000 kcal/m²·h°C, 5000 kcal/m²·h°C and 10000 kcal/m²·h°C. The next figure shows F.E. calculated stress histories. These results explain that the maximum stress increases and relaxes rapidly when heat transfer coefficient increases.
Fig.3.6 Green’s functions of pipes with various heat transfer coefficient (t=10mm)

When approximating stress histories of Fig.3.6 by Eq.(3.19), results have good agreement with original curves. Obtained factors by regression analysis are shown in Table 3.4 and in Fig.3.7. When heat transfer coefficient increases, values of factors increase and time constant becomes shorter.

Table 3.4 Coefficients of eigenfunctions for Pipes with various heat transfer coefficient

<table>
<thead>
<tr>
<th>$\tau_n$ sec</th>
<th>$t=10\text{mm}, h=2000\text{kcal}$</th>
<th>$t=10\text{mm}, h=5000\text{kcal}$</th>
<th>$t=10\text{mm}, h=10000\text{kcal}$</th>
</tr>
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<tbody>
<tr>
<td>10000</td>
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<td>0.0005</td>
</tr>
<tr>
<td>5000</td>
<td>0.0003</td>
<td>0.0019</td>
<td>-0.0013</td>
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<tr>
<td>2000</td>
<td>-0.0007</td>
<td>-0.0038</td>
<td>0.0025</td>
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<tr>
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<td>0.0012</td>
<td>0.0061</td>
<td>-0.0039</td>
</tr>
<tr>
<td>500</td>
<td>-0.0015</td>
<td>-0.0070</td>
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<tr>
<td>200</td>
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<td>-0.0067</td>
</tr>
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<tr>
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<td>-0.0252</td>
<td>-0.0523</td>
<td>-0.1546</td>
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</tbody>
</table>

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Fig. 3.7 Structural response characteristics to time constant (t=10mm)

To be compared, time when stress becomes the maximum and time constant when stress becomes the maximum stress divided by e were evaluated as in the next table and figure. This tendency agrees with Green function results. The latter can provide more detailed information such as the maximum stress level.

Table 3.5 Time constants for Pipes (t=10mm)

<table>
<thead>
<tr>
<th>Heat transfer coefficient kcal/m²h°C</th>
<th>1000</th>
<th>2000</th>
<th>5000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ max sec</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>τ 1/e sec</td>
<td>55</td>
<td>35</td>
<td>20</td>
<td>16</td>
</tr>
</tbody>
</table>
Fig. 3.8 Time constants of pipes (Wall thickness=10mm)
4. CONCLUSIONS

Structural response mechanism to fluid temperature fluctuation was clarified and was formulated by the relaxation functions. This function can quantify Green functions of structures. Green function enables fast calculations of stress response to arbitrary thermal transient conditions. Furthermore, we can obtain such characteristics of structures from Green function expressed by relaxation functions as the maximum stress level and time when stress becomes the maximum and time constant when stress becomes the maximum value divided by e.

For evaluation of the relaxation functions in structures, such evaluation methods were proposed as a modal analysis method, theoretical method and regression analysis method.
5. DISCUSSIONS

The modal analysis method requires more detailed formulation for programming. By using relaxation function, optimum design against thermal transient problems is expected.

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REFERENCES


